National Evaluation Series™

NES
PROFILE

Mathematics (304)

NES Profile: Mathematics (304)

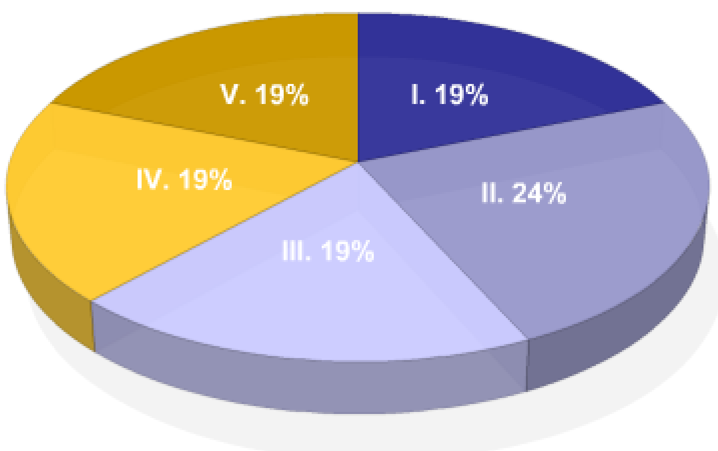
Overview

The resources below provide information about this test, including the approximate percentage of the total test score derived from each content domain. The complete set of the content domains, the test framework, is provided here and contains all of the competencies and descriptive statements that define the content of the test.

Select any of the content domains presented in the chart or its key to view:

- the test competencies associated with each content domain,
- a set of descriptive statements that further explain each competency,
- a sample test question aligned to each competency.

Test Field	Mathematics (304)
Test Format	Multiple-choice questions
Number of Questions	Approximately 150
Test Duration	Up to 4 hours and 15 minutes *(effective through September 11, 2016)* Up to 5 hours *(effective beginning September 12, 2016)*
Reference Materials	An on-screen scientific calculator is provided with your test. A formulas page is provided with your test. *Reference materials are provided on-screen as part of your test.*



Key	Approximate Percentage of Test	Content Domain	Range of Competencies
	19%	I. Mathematical Processes and Number Sense	0001–0003
	24%	II. Patterns, Algebra, and Functions	0004–0007
	19%	III. Measurement and Geometry	0008–0010
	19%	IV. Trigonometry and Calculus	0011–0013
	19%	V. Statistics, Probability, and Discrete Mathematics	0014–0016

SECONDARY MATHEMATICS FORMULAS

Formula	Description
$V = \frac{1}{3}Bh$	Volume of a right cone and a pyramid
$V = Bh$	Volume of a cylinder and prism
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$A = 2\pi rh + 2\pi r^2$	Surface area of a cylinder
$A = 4\pi r^2$	Surface area of a sphere
$A = \pi r\sqrt{r^2 + h^2} = \pi r\ell$	Lateral surface area of a right cone
$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + a_n)$	Sum of an arithmetic series
$S_n = \dfrac{a(1 - r^n)}{1 - r}$	Sum of a finite geometric series
$\sum_{n=0}^{\infty} ar^n = \dfrac{a}{1 - r},\ \lvert r \rvert < 1$	Sum of an infinite geometric series
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$	Law of sines
$c^2 = a^2 + b^2 - 2ab \cos C$	Law of cosines
$(x - h)^2 + (y - k)^2 = r^2$	Equation of a circle
$(y - k) = 4c(x - h)^2$	Equation of a parabola
$\dfrac{(x - h)^2}{a^2} + \dfrac{(y - k)^2}{b^2} = 1$	Equation of an ellipse
$\dfrac{(x - h)^2}{a^2} - \dfrac{(y - k)^2}{b^2} = 1$	Equation of a hyperbola

Calculator Information

A scientific calculator will be provided with your test. You may not use your own scientific calculator or calculator manual.

Content Domain I: Mathematical Processes and Number Sense

Competencies:

0001 Understand mathematical problem solving.

Descriptive Statements:

- Identify an appropriate problem-solving strategy for a particular problem.
- Analyze the use of estimation in a variety of situations (e.g., rounding, area, plausibility).
- Solve mathematical and real-world problems involving integers, fractions, decimals, and percents.
- Solve mathematical and real-world problems involving ratios, proportions, and average rates of change.

Sample Item:

In four half-cup samples of a cereal containing dried cranberries, the numbers of cranberries were 17, 22, 22, and 18. Nutrition information on a box of this cereal defines the serving size as 1 cup or 53 grams. If a box contains 405 grams, which of the following is the best estimate of the number of cranberries in one box of this cereal?

A. less than 300

B. between 300 and 325

C. between 326 and 350

D. more than 350

Correct Response and Explanation

B. This question requires the examinee to analyze the use of estimation in a variety of situations (e.g., rounding, area, plausibility). There are approximately 20 cranberries per $\frac{1}{2}$ cup, or 40 cranberries per cup. The number of cups in a box is $405 \div 53$, which is approximately equal to $400 \div 50 = 8$ (rounding both the numerator and denominator down minimizes the error). Thus the approximate number of cranberries in a box of this cereal is $8 \times 40 = 320$, which is within the interval of response B.

0002 Understand mathematical communication, connections, and reasoning.

Descriptive Statements:

- Translate between representations (e.g., graphic, verbal, symbolic).
- Recognize connections between mathematical concepts.
- Analyze inductive and deductive reasoning.
- Apply principles of logic to solve problems.
- Demonstrate knowledge of the historical development of major mathematical concepts, including contributions from diverse cultures.

Sample Item:

Given statements *p* and *q*, which of the following is the truth table for the compound statement $p \leftrightarrow (q \vee {\sim}p)$?

A.

p	q	$p \leftrightarrow (q \vee {\sim}p)$
T	T	T
T	F	F
F	T	F
F	F	F

B.

p	q	$p \leftrightarrow (q \vee {\sim}p)$
T	T	T
T	F	F
F	T	T
F	F	F

C.

p	q	$p \leftrightarrow (q \vee {\sim}p)$
T	T	T
T	F	F
F	T	T
F	F	T

D.

p	q	$p \leftrightarrow (q \vee {\sim}p)$
T	T	T
T	F	F
F	T	F
F	F	T

Correct Response and Explanation

A. This question requires the examinee to apply principles of logic to solve problems. First a truth value column for ${\sim}p$ is computed as F, F, T, T (in vertical order). Then this column is used to compute truth values for the statement $q \vee {\sim}p$: T, F, T, T. Finally, this result is used to compute truth values for the full expression $p \leftrightarrow (q \vee {\sim}p)$: T, F, F, F.

0003 Understand number theory.

Descriptive Statements:

- Analyze the group structure of the real numbers.

- Use complex numbers and their operations.
- Analyze the properties of numbers and operations.
- Apply the principles of basic number theory (e.g., prime factorization, greatest common factor, least common multiple).

Sample Item:

If *p* and *q* are prime numbers and $\frac{4}{q^3} = \frac{p^2}{50}$ what is the value of ($p + q$)?

A. 5

B. 7

C. 8

D. 9

Correct Response and Explanation

B. This question requires the examinee to apply the principles of basic number theory (e.g., prime factorization, greatest common factor, least common multiple). The variables can be isolated by multiplying both sides of the equation by $50q^3$, which yields $200 = p^2q^3$. If p and q are both prime, then p^2q^3 is the prime factorization of 200. Since $200 = 25 \times 8 = 5^2 \times 2^3$, and 5 and 2 are both primes, p must be 5 and q must be 2, so $p + q = 5 + 2 = 7$.

Content Domain II: Patterns, Algebra, and Functions

Competencies:

0004 Understand relations and functions.

Descriptive Statements:

- Demonstrate knowledge of relations and functions and their applications.
- Perform operations with functions, including compositions and inverses.
- Analyze characteristics of functions.
- Interpret different representations of functions.

Sample Item:

Which of the following equations represents the inverse of $y = \dfrac{6x - 4}{1 + 3x}$?

A. $y = \dfrac{x - 4}{3x + 6}$

B. $y = \dfrac{x + 4}{6 - 3x}$

C. $y = \dfrac{1 + 3x}{6x - 4}$

D. $y = \dfrac{1 - 3x}{6x + 4}$

Correct Response and Explanation

B. This question requires the examinee to perform operations with functions, including compositions and inverses. To find the inverse of a function of the form $y = f(x)$, the original equation is rearranged by solving it for x as a function of y: $y = \dfrac{6x - 4}{1 + 3x} \Rightarrow y(1 + 3x) = 6x - 4 \Rightarrow y + 3xy = 6x - 4 \Rightarrow y + 4 = 6x - 3xy \Rightarrow y + 4 = x(6 - 3y) \Rightarrow x = \dfrac{y + 4}{6 - 3y}$. Exchanging the variables x and y results in the inverse function f^{-1}, $y = \dfrac{x + 4}{6 - 3x}$

0005 Understand linear, quadratic, and higher-order polynomial functions.

Descriptive Statements:

- Analyze the relationship between a linear, quadratic, or higher-order polynomial function and its graph.
- Solve linear and quadratic equations and inequalities using a variety of methods.
- Solve systems of linear equations or inequalities using a variety of methods.
- Solve higher-order polynomial equations and inequalities in one and two variables.
- Analyze the characteristics of linear, quadratic, and higher-order polynomial equations.
- Analyze real-world problems involving linear, quadratic, and higher-order polynomial functions.

Sample Item:

	Order 1	Order 2	Order 3
soft drink	4	6	3
large pizza	1	2	1
garlic bread	1	1	0
Total Cost	$19.62	$34.95	$16.50

Given the table of orders and total costs above, and that there is a solution to the problem, which of the following matrix equations could be used to find *d*, *p*, and *g*, the individual prices for a soft drink, a large pizza, and garlic bread respectively?

A.
$$\begin{bmatrix} 4 & 6 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 19.62 \\ 34.95 \\ 16.50 \end{bmatrix} = \begin{bmatrix} d \\ p \\ g \end{bmatrix}$$

B.
$$\begin{bmatrix} 4 & 6 & 3 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} d \\ p \\ g \end{bmatrix} = \begin{bmatrix} 19.62 \\ 34.95 \\ 16.50 \end{bmatrix}$$

C.
$$\begin{bmatrix} 4 & 1 & 1 \\ 6 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix}\begin{bmatrix} 19.62 \\ 34.95 \\ 16.50 \end{bmatrix} = \begin{bmatrix} d \\ p \\ g \end{bmatrix}$$

D.
$$\begin{bmatrix} 4 & 1 & 1 \\ 6 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix}\begin{bmatrix} d \\ p \\ g \end{bmatrix} = \begin{bmatrix} 19.62 \\ 34.95 \\ 16.50 \end{bmatrix}$$

Correct Response and Explanation

D. This question requires the examinee to solve systems of linear equations or inequalities using a variety of methods. The system of linear equations can be solved using matrices. Each order can be expressed as an equation, with all three equations written with the variables in the same sequence. The first order is represented by the equation $4d + p + g = 19.62$, the second order by $6d + 2p + g = 34.95$, and the third order by $3d + p = 16.50$. The rows of the left-hand matrix contain the coefficients of d, p, and g for each equation: (4 1 1), (6 2 1), and (3 1 0). The middle matrix contains the variables, d, p, g. The right-hand matrix vertically arranges the constants of the equations.

0006 Understand exponential and logarithmic functions.

Descriptive Statements:

- Apply the laws of exponents and logarithms.
- Analyze the relationship between exponential and logarithmic functions.
- Analyze exponential and logarithmic functions and their graphs.
- Analyze real-world problems involving exponential and logarithmic functions.

Sample Item:

Which of the following is equivalent to the equation $3 \log_{10} x - 2 \log_{10} y = 17$?

A. $3x - 2y = 10^{17}$

B. $x^3 - y^2 = 10^{17}$

C. $\frac{x^3}{y^2} = 10^{17}$

D. $\frac{3x}{2y} = 10^{17}$

Correct Response and Explanation

C. This question requires the examinee to apply the laws of exponents and logarithms.

$N\log_a M = \log_a M^N \Rightarrow 3\log_{10} x = \log_{10} x^3$ and $2\log_{10} y = \log_{10} y^2$.

$\text{Log}_a M - \log_a N \Rightarrow \log_a \frac{M}{N} \Rightarrow \log_{10} x^3 - \log_{10} y^2 = \log_{10} \frac{x^3}{y^2}$.

Since $\log_a M = N$ is equivalent to $a^N = M$, then $\log_{10} \frac{x^3}{y^2} = 17$ is equivalent to $10^{17} = \frac{x^3}{y^2}$.

0007 Understand rational, radical, absolute value, and piece-wise defined functions.

Descriptive Statements:

- Manipulate rational, radical, and absolute value expressions, equations, and inequalities.
- Analyze the relationship between a rational, radical, absolute value, or piece-wise defined function and its graph.
- Analyze rational, radical, absolute value, and piece-wise defined functions in terms of domain, range, and asymptotes.
- Analyze real-world problems involving rational, radical, absolute value, and piece-wise defined functions.

Sample Item:

Which of the following represents the domain of the function $f(x) = \dfrac{\sqrt{2x+3}}{3x+1}$?

A. $\left(-\frac{3}{2}, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$

B. $\left[-\frac{3}{2}, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$

C. $\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, \infty\right)$

D. $\left[-\frac{3}{2}, -\frac{1}{3}\right) \cup \left(-\frac{1}{3}, 0\right) \cup (0, \infty)$

Correct Response and Explanation

B. This question requires the examinee to analyze rational, radical, absolute value, and piece-wise defined functions in terms of domain, range, and asymptotes. Unless otherwise specified, the domain of a function is the range of values for which the function has a real number value. A rational function must have a nonzero denominator, and solving the equation $3x + 1 = 0$ yields $x = -\frac{1}{3}$. Thus this value must be excluded from the domain. The radical expression in the numerator must have a non-negative argument and solving the inequality $2x + 3 \geqslant 0$ yields $x \geqslant -\frac{3}{2}$. Putting these two results together results in $-\frac{3}{2} \leqslant x < -\frac{1}{3}$ or $x > -\frac{1}{3}$. The "or" represents the union of the two sets defined by the inequalities, or the union of the two intervals.

Content Domain III: Measurement and Geometry

Competencies:

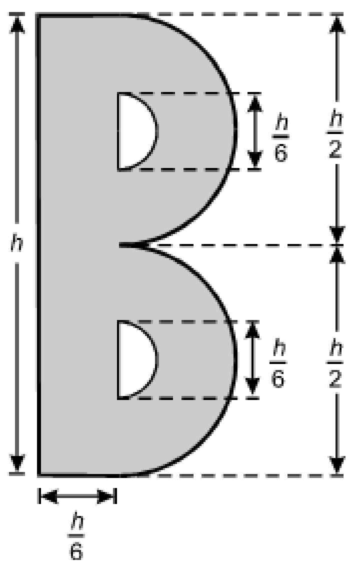
0008 Understand measurement principles and procedures.

Descriptive Statements:

- Analyze the use of various units and unit conversions within the customary and metric systems.
- Apply the concepts of similarity, scale factors, and proportional reasoning to solve measurement problems.
- Analyze precision, error, and rounding in measurements and computed quantities.
- Apply the concepts of perimeter, circumference, area, surface area, and volume to solve real-world problems.

Sample Item:

The shape of the letter B is designed as shown below, consisting of rectangles and semicircles.



Which of the following formulas gives the area *A* of the shaded region as a function of its height *h*?

A. $A = h^2\left(\frac{1}{6} + \frac{\pi}{18}\right)$

B. $A = h^2\left(\frac{1}{6} + \frac{\pi}{9}\right)$

C. $A = h^2\left(\frac{1}{6} + \frac{2\pi}{9}\right)$

D. $A = h^2\left(\frac{1}{6} + \frac{5\pi}{18}\right)$

Correct Response and Explanation

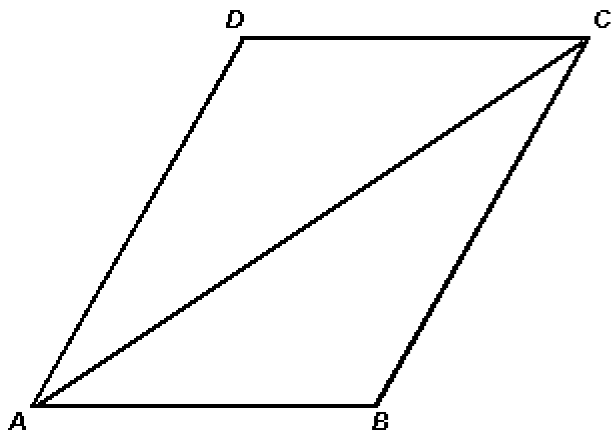
A. This question requires the examinee to apply the concepts of perimeter, circumference, area, surface area, and volume to solve real-world problems. The total area of the letter B can be viewed as the area of an $h \times \frac{h}{6}$ rectangle plus the area of a circle with radius $\frac{h}{4}$ minus the area of a circle with radius $\frac{h}{12}$, or $\frac{h^2}{6} + \pi\left(\frac{h}{4}\right)^2 - \pi\left(\frac{h}{12}\right)^2$. This simplifies to $\frac{h^2}{6} + \frac{\pi h^2}{16} - \frac{\pi h^2}{144}$ and further to $h^2\left(\frac{1}{6} + \frac{\pi}{16} - \frac{\pi}{144}\right)$ and $h^2\left(\frac{1}{6} + \frac{9\pi}{144} - \frac{\pi}{144}\right)$ and $h^2\left(\frac{1}{6} + \frac{\pi}{18}\right)$.

0009 Understand Euclidean geometry in two and three dimensions.

Descriptive Statements:

- Demonstrate knowledge of axiomatic systems and of the axioms of non-Euclidean geometries.
- Use the properties of polygons and circles to solve problems.
- Apply the Pythagorean theorem and its converse.
- Analyze formal and informal geometric proofs, including the use of similarity and congruence.
- Use nets and cross sections to analyze three-dimensional figures.

Sample Item:



Given: $\overline{AB} \parallel \overline{DC}$; $\overline{AB} \cong \overline{DC}$

Prove: $\triangle ABC \cong \triangle CDA$

Statement	Reason
1. $\overline{AB} \cong \overline{DC}$	Given
2.	
3. $\overline{AB} \parallel \overline{DC}$	Given
4.	
5. $\triangle ABC \cong \triangle CDA$	________

In the proof above, steps 2 and 4 are missing. Which of the following reasons justifies step 5?

A. AAS

B. ASA

C. SAS

D. SSS

Correct Response and Explanation

C. This question requires the examinee to analyze formal and informal geometric proofs, including the use of similarity and congruence. The side-angle-side (SAS) theorem can be used to show that ΔABC and ΔCDA are congruent if each has two sides and an included angle that are congruent with two sides and an included angle of the other. In the diagram $\overline{AB}$ and $\overline{DC}$ are given as congruent, and the missing statement 2 is that $\overline{AC}$ is congruent to itself by the reflexive property of equality. The included angles $\angle BAC$ and $\angle DCA$ are congruent because they are alternate interior angles constructed by the transversal $\overline{AC}$ that crosses the parallel line segments $\overline{AB}$ and $\overline{DC}$. Thus ΔABC and ΔCDA meet the requirements for using SAS to prove congruence.

0010 Understand coordinate and transformational geometry.

Descriptive Statements:

- Analyze two- and three-dimensional figures using coordinate systems.
- Apply concepts of distance, midpoint, and slope to classify figures and solve problems in the coordinate plane.
- Analyze conic sections.
- Determine the effects of geometric transformations on the graph of a function or relation.
- Analyze transformations and symmetries of figures in the coordinate plane.

Sample Item:

The vertices of triangle *ABC* are *A*(–5, 3), *B*(2, 2), and *C*(–1, –5). Which of the following is the length of the median from vertex *B* to side *AC?*

A. 4

B. $2\sqrt{5}$

C. $\sqrt{34}$

D. $4\sqrt{5}$

Correct Response and Explanation

C. This question requires the examinee to apply concepts of distance, midpoint, and slope to classify figures and solve problems in the coordinate plane. The midpoint of side *AC* where its median intersects is computed as $\left(\frac{-5+-1}{2}, \frac{3+-5}{2}\right)$ = (–3, –1). The distance from *B*(2, 2) to (–3, –1) is computed as $d = \sqrt{(-3-2)^2 + (-1-2)^2} = \sqrt{25+9} = \sqrt{34}$.

Content Domain IV: Trigonometry and Calculus

Competencies:

0011 Understand trigonometric functions.

Descriptive Statements:

- Apply trigonometric functions to solve problems involving distance and angles.
- Apply trigonometric functions to solve problems involving the unit circle.
- Manipulate trigonometric expressions and equations using techniques such as trigonometric identities.
- Analyze the relationship between a trigonometric function and its graph.
- Use trigonometric functions to model periodic relationships.

Sample Item:

Which of the following are the solutions to $2 \sin^2 \theta = \cos \theta + 1$ for $0 < \theta \leqslant 2\pi$?

A. $\frac{\pi}{6}, \pi, \frac{11\pi}{6}$

B. $\frac{\pi}{3}, \pi, \frac{5\pi}{3}$

C. $\frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$

D. $\frac{5\pi}{6}, \frac{7\pi}{6}, 2\pi$

Correct Response and Explanation

B. This question requires the examinee to manipulate trigonometric expressions and equations using techniques such as trigonometric identities. Since $\sin^2 \theta = 1 - \cos^2 \theta$, $2 \sin^2 \theta = \cos \theta + 1 \Rightarrow 2(1 - \cos^2 \theta) = \cos \theta + 1 \Rightarrow 2 \cos^2 \theta + \cos \theta - 1 = 0 \Rightarrow (2 \cos \theta - 1)(\cos \theta + 1) = 0 \Rightarrow \cos \theta = \frac{1}{2}$ or $\cos \theta = -1$. Thus for $0 < \theta \leqslant 2\pi$, $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$, or π.

0012 Understand differential calculus.

Descriptive Statements:

- Evaluate limits.
- Demonstrate knowledge of continuity.
- Analyze the derivative as the slope of a tangent line and as the limit of the difference quotient.
- Calculate the derivatives of functions (e.g., polynomial, exponential, logarithmic).
- Apply differentiation to analyze the graphs of functions.
- Apply differentiation to solve real-world problems involving rates of change and optimization.

Sample Item:

If $f(x) = 3x^4 - 8x^2 + 6$, what is the value of $\lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$?

A. –4

B. –1

C. 1

D. 4

Correct Response and Explanation

A. This question requires the examinee to analyze the derivative as the slope of a tangent line and as the limit of the difference quotient. The limit expression is equivalent to the derivative $f'(1)$. Since it is much easier to evaluate the derivative of a polynomial, this is preferred over evaluating the limit expression. $f'(x) = 12x^3 - 16x$, so $f'(1) = 12 - 16 = -4$.

0013 Understand integral calculus.

Descriptive Statements:

- Analyze the integral as the area under a curve and as the limit of the Riemann sum.
- Calculate the integrals of functions (e.g., polynomial, exponential, logarithmic).
- Apply integration to analyze the graphs of functions.
- Apply integration to solve real-world problems.

Sample Item:

A sum of $2000 is invested in a savings account. The amount of money in the account in dollars after t years is given by the equation $A = 2000^{e0.05t}$. What is the approximate average value of the account over the first two years?

A. $2103

B. $2105

C. $2206

D. $2210

Correct Response and Explanation

A. This question requires the examinee to apply integration to solve real-world problems. The average value of a continuous function $f(x)$ over an interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$. Since the independent variable t represents the number of years, the average daily balance over 2 years will be $\frac{1}{2}$ of the integral of the function evaluated from 0 to 2: $\frac{1}{2} \cdot \int_0^2 2000e^{0.05t}dt = \frac{1000}{0.05}(e^{0.1} - 1) \approx 2103$.

Content Domain V: Statistics, Probability, and Discrete Mathematics

Competencies:

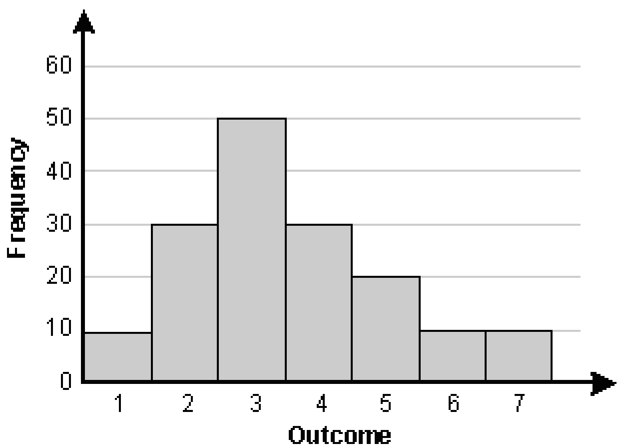
0014 Understand principles and techniques of statistics.

Descriptive Statements:

- Use appropriate formats for organizing and displaying data.
- Analyze data in a variety of representations.
- Analyze the use of measures of central tendency and variability.
- Analyze the effects of bias and sampling techniques.

Sample Item:

Which of the following statements describes the set of data represented by the histogram below?



A. The mode is equal to the mean.

B. The mean is greater than the median.

C. The median is greater than the range.

D. The range is equal to the mode.

Correct Response and Explanation

B. This question requires the examinee to analyze data in a variety of representations. The mean can be calculated as [10(1) + 30(2) + 50(3) + 30(4) + 20(5) + 10(6) + 10(7)] ÷ 160 = 3.5625. The median is the 50th percentile, which is 3. The mode is the most frequent value, which is 3. The range is 7 – 1 = 6. Thus "the mean is greater than the median" is the correct response.

0015 Understand principles and techniques of probability.

Descriptive Statements:

- Determine probabilities of simple and compound events and conditional probabilities.
- Use counting principles to calculate probabilities.

- Use a variety of graphical representations to calculate probabilities.
- Select simulations that model real-world events.
- Analyze uniform, binomial, and normal probability distributions.

Sample Item:

The heights of adults in a large group are approximately normally distributed with a mean of 65 inches. If 20% of the adult heights are less than 62.5 inches, what is the probability that a randomly chosen adult from this group will be between 62.5 inches and 67.5 inches tall?

A. 0.3

B. 0.4

C. 0.5

D. 0.6

Correct Response and Explanation

D. This question requires the examinee to analyze uniform, binomial, and normal probability distributions. A normal distribution is symmetric about the mean. Thus if 20% of the heights are less than 62.5 inches (2.5 inches from the mean), then 20% of the heights will be greater than 67.5 inches (also 2.5 inches from the mean). Thus 100% – (20% + 20%) = 60% and the probability is 0.6 that the adult will be between 62.5 and 67.5 inches tall.

0016 Understand principles of discrete mathematics.

Descriptive Statements:

- Apply concepts of permutations and combinations to solve problems.
- Analyze sequences and series including limits and recursive definitions.
- Perform operations on matrices and vectors.
- Apply set theory to solve problems.

Sample Item:

Five different algebra textbooks, two different calculus textbooks, and four different geometry textbooks are to be arranged on a shelf. How many different arrangements are possible if the books of each subject must be kept together?

A. $(5 \cdot 2 \cdot 4)^3$

B. $\dfrac{11!}{5!\,2!\,4!}$

C. $5!\,2!\,4!\,3!$

D. $\dfrac{11^3}{(5!\,2!\,4!)^3}$

Correct Response and Explanation

C. This question requires the examinee to apply concepts of permutations and combinations to solve problems. If the books in each of the 3 subjects must be kept together, then the number of ways the groups of books can be arranged by subject is represented by 3!. If there are *n* books within a subject, the number of ways the books can be arranged is *n*!. Thus the algebra books can be arranged in 5! different ways, the calculus books can be arranged in 2! different ways, and the geometry books can be arranged in 4! different ways. Since there is independence between the different arrangements computed, the total number of ways the books can be arranged is the product of all the factorials 3! 5! 2! 4! which is equivalent to 5! 2! 4! 3!.