

New Mexico

Teacher Assessments™

Study Guide

New Mexico Content Knowledge Assessments™

24 Middle Level Mathematics



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An Overview of the Testing Program, How to Prepare for the Assessments, and the Day of the Test: Helpful Hints

The first three sections of the study guide are available in a separate PDF file. Click the link below to view or print these sections.

[An Overview of the Testing Program, How to Prepare for the Assessments, and the Day of the Test:
Helpful Hints](#)



An Introduction to the New Mexico Content Knowledge Assessment of Middle Level Mathematics

The New Mexico Content Knowledge Assessments are designed to measure the subject matter knowledge and skills that are needed to teach effectively in New Mexico schools. This assessment is intended as an option for elementary K–8 teachers to establish that they are “highly qualified” under the “No Child Left Behind” legislation. The Middle Level Mathematics test may not be taken to obtain a secondary license endorsement.

The Middle Level Mathematics test consists of 100 selected-response questions.



Test Framework

Mathematical Reasoning, Problem Solving, and Methods, Number Concepts,
and the Historical Development of Mathematics
Number Systems, Number Concepts, and Number Theory
Geometry and Measurement
Data Analysis, Statistics, and Probability
Patterns, Algebraic Relationships, and Functions

SUBAREA I—MATHEMATICAL REASONING, PROBLEM SOLVING, AND METHODS, NUMBER CONCEPTS, AND THE HISTORICAL DEVELOPMENT OF MATHEMATICS

0001 Understand and apply problem-solving strategies, connections among different mathematical ideas, and mathematical modeling to solve application problems encountered in life.

For example:

- analyze information to define problems appropriately
- recognize appropriate strategies (e.g., drawing a diagram, working backward, inductive reasoning, creating a simpler problem) to solve problems
- select an appropriate tool, technology, or mathematical principle to solve a given problem
- recognize connections among different mathematical concepts
- apply mathematics across the curriculum and in everyday contexts

0002 Understand principles of mathematical reasoning and techniques for communicating mathematical ideas.

For example:

- use appropriate mathematical terminology and symbols
- communicate mathematical ideas and concepts using a variety of numeric, symbolic, and graphic methods
- make connections among numeric, symbolic, graphic, and verbal representations of mathematical ideas and concepts
- distinguish between inductive and deductive reasoning
- draw valid conclusions from stated conditions

0003 Understand the development of mathematical concepts, skills, and applications from their origins to current times, including the role of technology.

For example:

- demonstrate knowledge of the historical development of major mathematical concepts, including contributions made by various cultures and individuals
- recognize appropriate applications and limitations of contemporary technologies such as computing tools and graphing calculators for defining and solving problems
- solve problems using scientific calculators and simple computer algorithms

SUBAREA II—NUMBER SYSTEMS, NUMBER CONCEPTS, AND NUMBER THEORY

0004 Understand number systems, number theory, and ways of representing numbers.

For example:

- analyze the role of counting, grouping, and place value in ancient and modern numeration systems
- analyze properties of prime numbers, factors, multiples, and divisibility
- use a variety of equivalent representations of numbers (e.g., fractions, decimals, percents, roots, exponents)
- use scientific notation to solve problems involving very large and very small numbers
- demonstrate an understanding of the use of manipulatives, verbal expressions, and geometric models to represent numbers and numerical operations

0005 Understand operations on numbers and properties of number operations.

For example:

- apply properties of numbers and number operations, including the commutative and distributive properties, to manipulate and simplify algebraic expressions
- solve a variety of problems using number operations, including ratio and proportion problems
- demonstrate an understanding of inverse operations, identities, and algebraic operations with real numbers and variables
- estimate solutions and evaluate reasonableness of estimation strategies and solutions
- compute fluently using fractions, decimals, and integers

SUBAREA III—GEOMETRY AND MEASUREMENT

0006 Apply geometric concepts and reasoning, including two- and three-dimensional coordinate geometry.

For example:

- compare and analyze shapes and formally establish the relationships among them (e.g., congruence, similarity)
- solve problems involving parallelism, perpendicularity, congruence, similarity, translation, reflection, rotation, dilation, tessellation, and symmetry on a plane
- apply the Pythagorean theorem to solve problems
- understand and apply coordinate geometry formulas and concepts, including slope, distance formula, midpoint formula, and parallel and perpendicular lines
- understand properties of geometric shapes, including circles and polygons, and solve problems involving these properties

0007 Understand and use measurement.

For example:

- solve problems using appropriate units of measurement for angles (e.g., degree, radian), length, area, volume, mass, temperature, time, density, pressure, and rates of change
- use standard and nonstandard units of measurement to an appropriate degree of accuracy
- identify appropriate tools for performing measurements
- convert measurements within and between traditional and metric measuring systems
- derive and apply formulas for the perimeter, area, surface area, or volume of two- and three-dimensional composite figures
- solve problems using right triangle trigonometry (e.g., problems involving indirect measurements)

SUBAREA IV—DATA ANALYSIS, STATISTICS, AND PROBABILITY

0008 Understand methods of collecting, organizing, displaying, describing, and analyzing data.

For example:

- formulate questions and collect data from life experiences using appropriate techniques in a variety of situations
- analyze and interpret data and formulate statistical conclusions based on the data
- use and interpret a variety of charts and graphs to organize and display data (e.g., pie charts, box plots, stem-and-leaf plots, scatter plots, frequency histograms) appropriately
- apply and interpret measures of central tendency (e.g., mean, median, mode) and spread (e.g., range, standard deviation)

0009 Understand the theory of probability and probability distributions.

For example:

- enumerate the sample space of an event and determine simple and compound probabilities
- find the probability of dependent and independent events and determine conditional probabilities
- use simulations and sampling to determine experimental probabilities
- solve problems using geometric probability (e.g., ratio of two areas)
- solve simple problems involving probability, inference, and the testing of hypotheses
- use simple combinations and permutations to solve counting problems

SUBAREA V—PATTERNS, ALGEBRAIC RELATIONSHIPS, AND FUNCTIONS

0010 Describe, analyze, and generalize mathematical patterns.

For example:

- recognize, describe, and extend a variety of numerical and geometric patterns
- represent and record patterns using manipulatives, tables, graphs, and matrices
- analyze and generalize sequences and series (e.g., Fibonacci, geometric) and use them to solve problems
- recognize and extend recursive patterns and use them to solve problems
- use patterns and relationships to make inferences, predictions, and decisions

0011 Use variables and symbolic expressions to describe and analyze patterns of change, functions, and relationships among variables.

For example:

- represent situations using variables, expressions, and open sentences
- manipulate and simplify symbolic expressions
- explore patterns of change characteristic of families of functions (e.g., linear, quadratic, exponential)
- analyze functions in terms of range, domain, intercepts, operations, compositions, and inverses
- analyze the graphs of functions

0012 Understand properties and applications of linear, quadratic, exponential, and trigonometric functions and solve related equations and inequalities.

For example:

- analyze the relationship between a linear equation and its graph, including its slope and intercepts
- predict or interpolate values using a linear model
- model and solve problems using linear equations, inequalities, and systems of linear equations
- analyze the roots and properties of a quadratic equation
- solve quadratic equations, inequalities, and systems
- analyze polynomial functions and their graphs and use them to solve problems (e.g., zeros, minima and maxima, symmetry)
- model and solve problems involving exponential functions
- examine the relationship between the trigonometric functions and their inverses and between the trigonometric and circular functions



Sample Test Directions

A sample of the general directions for the New Mexico Content Knowledge Assessment of Middle Level Mathematics is shown in the box below.

You should have in front of you:

- (1) a test booklet for the assessment for which you registered (check the assessment name on the front cover);
- (2) an Answer Sheet A; and
- (3) a No. 2 lead pencil.

Note: For the **Middle Level Mathematics (24)** test only, you may use your own calculator if it is an approved model.

IF YOU ARE MISSING ANY OF THESE MATERIALS, NOTIFY YOUR TEST ADMINISTRATOR. REMOVE ALL OTHER MATERIALS FROM YOUR DESK.

TEST DIRECTIONS

Each question in this booklet is a selected-response question with four answer choices. Read each question carefully and choose the ONE best answer. Record your answer on Answer Sheet A in the space that corresponds to the question number. Completely fill in the circle having the same letter as the answer you have chosen. *Use only a No. 2 lead pencil.*

- Sample Question:
1. What is the capital of New Mexico?
 - A. Albuquerque
 - B. Las Cruces
 - C. Santa Fe
 - D. Silver City

The correct answer to this question is C. You would indicate that on Answer Sheet A as follows:

1. A B C D

Try to answer all questions. Even if you are unsure of an answer, it is better to guess than not to answer a question at all. You will NOT be penalized for guessing.

You may use the margins of the test booklet for scratch paper, but all of your answers must be recorded on the answer sheet. Answers that are in the test booklet will not be scored.

The words "End of Test" indicate that you have completed the test. You may go back and review your work, and be sure you have answered all questions before raising your hand for dismissal. Your test materials must be returned to a test administrator when you finish the test.

FOR TEST SECURITY REASONS, YOU MAY NOT TAKE NOTES OR REMOVE ANY OF THE TEST MATERIALS FROM THE ROOM.

This testing session will last four hours. You may work at your own pace. If you have any questions, please ask them now before beginning the test.



DO NOT GO ON UNTIL YOU ARE TOLD TO DO SO.



Sample Selected-Response Questions

This section presents sample selected-response questions for you to review as part of your preparation for the New Mexico Content Knowledge Assessment of Middle Level Mathematics. To demonstrate how the test competencies may be assessed, each sample question is preceded by the competency that it measures. On an actual test, the competencies will not be given.

The sample selected-response questions are designed to illustrate the nature of the test questions. They should not be used as a diagnostic tool to determine your individual strengths and weaknesses. The selected-response questions require you to demonstrate more than the ability to recall factual information. They ask you to think critically about the information, to analyze it, to consider it carefully, or to apply it to a hypothetical situation.

Work through each question carefully before referring to the answer key, which is located at the end of the section.

Competency 0002

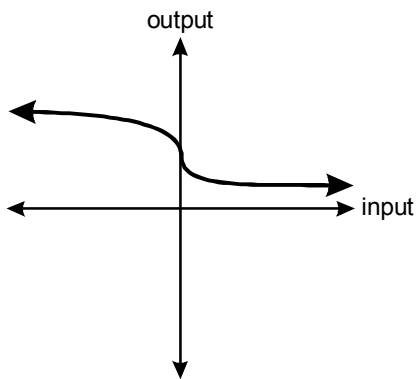
Understand principles of mathematical reasoning and techniques for communicating mathematical ideas.

1. A function is described by the following process.

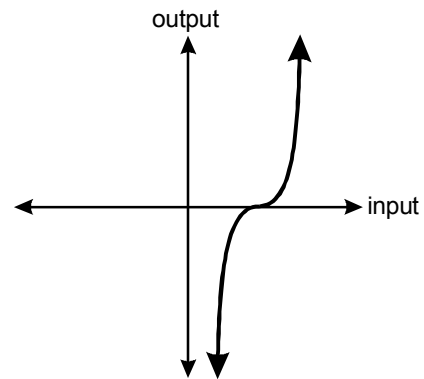
1. Double the input value.
2. Subtract four.
3. Take the cube root.

Which of the following graphs best represents this function?

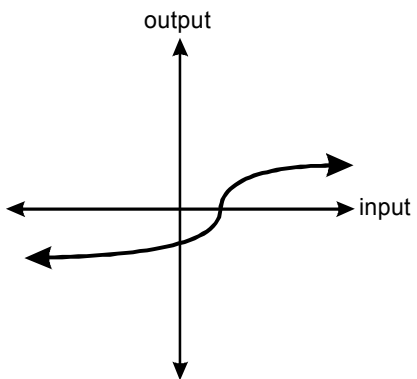
A.



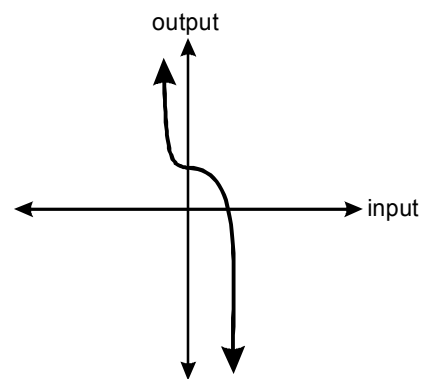
B.



C.



D.



Competency 0003

Understand the development of mathematical concepts, skills, and applications from their origins to current times, including the role of technology.

2. Use the information below to answer the question that follows.

Add 4 to 100, multiply by 8, and add 62,000. The result is approximately the circumference of a circle of which the diameter is 20,000.

This problem, which appears in a text from India written in 499 A.D., implies that the value of π is approximately which of the following?

- A. 3.2
- B. 3.14
- C. 3.1416
- D. 3.142857

Competency 0004

Understand number systems, number theory, and ways of representing numbers.

3. The number $x^{0.45}$ is equivalent to which of the following?

- A. $\sqrt[0.45]{x}$
- B. $\sqrt[20]{x^9}$
- C. $\frac{1}{x^{0.45}}$
- D. $\log_{0.45} x$

Competency 0005

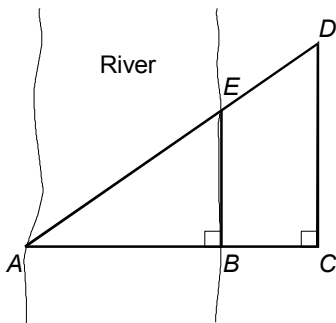
Understand operations on numbers and properties of number operations.

4. A solution of salt and water is 80% water by weight. After a period of time, some of the water evaporates and the solution is 40% water by weight. What is the ratio of the final weight of water to the initial weight of water in the mixture?
- A. $\frac{1}{6}$
- B. $\frac{2}{10}$
- C. $\frac{1}{3}$
- D. $\frac{1}{2}$

Competency 0006

Apply geometric concepts and reasoning, including two- and three-dimensional coordinate geometry.

5. Use the diagram below to answer the question that follows.



The diagram above shows a method for estimating AB , the width of a river. If $BC = 30$ m, $CD = 80$ m, and $BE = 60$ m, what is the width of the river?

- A. 22.5 m
- B. 40 m
- C. 90 m
- D. 160 m

Competency 0007

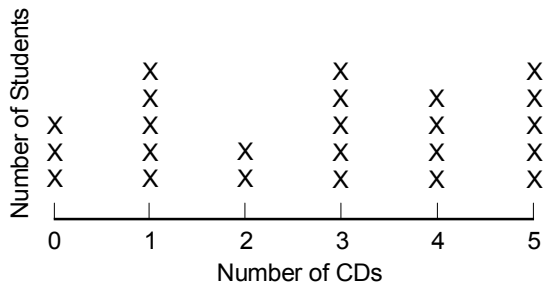
Understand and use measurement.

6. A right cylindrical container with a radius of 4 cm and a height of 6 cm has a leak in the bottom. The container is initially filled with water. After 1.5 hours the container is three-fourths full. At what rate is water leaking from the container?
- A. $\frac{2}{45}\pi$ cm³/min
- B. $\frac{4}{15}\pi$ cm³/min
- C. $\frac{11}{15}\pi$ cm³/min
- D. $\frac{8}{5}\pi$ cm³/min

Competency 0008

Understand methods of collecting, organizing, displaying, describing, and analyzing data.

7. Use the diagram below to answer the question that follows.



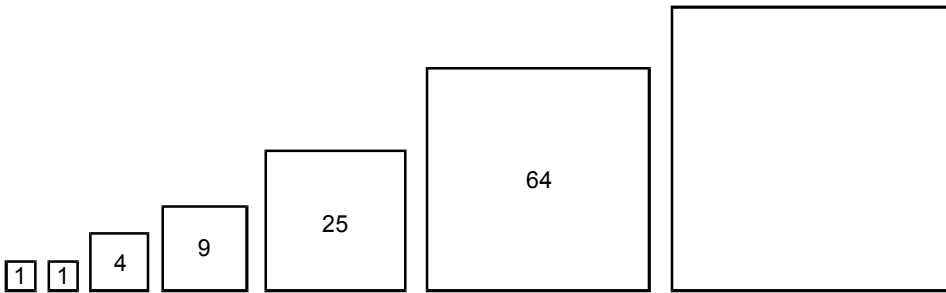
The diagram above shows the number of music CDs owned by a sample of students in a middle school. What is the mean number of CDs owned by the students?

- A. 2.50
- B. 2.71
- C. 2.83
- D. 3.40

Competency 0010

Describe, analyze, and generalize mathematical patterns.

8. Use the diagram below to answer the question that follows.



Six of the squares in the diagram above have the areas indicated. What is the area of the seventh square?

- A. 81
- B. 100
- C. 144
- D. 169

Competency 0011

Use variables and symbolic expressions to describe and analyze patterns of change, functions, and relationships among variables.

9. Which of the following is equivalent to the expression

$$\frac{x+2}{x+4} + \frac{x}{x+3} ?$$

- A. $\frac{2x+2}{2x+7}$
- B. $\frac{2x^2+10}{x^2+12}$
- C. $\frac{2x^2+5x+10}{x^2+7x+12}$
- D. $\frac{2x^2+9x+6}{x^2+7x+12}$

Competency 0012

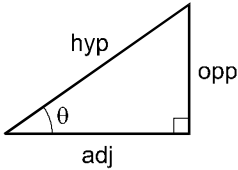
Understand properties and applications of linear, quadratic, exponential, and trigonometric functions and solve related equations and inequalities.

10. A scientist observes that the population of bacteria cells in a culture doubles every 5 hours. If the initial population is 10,000 cells, which of the following equations could be used to predict the number of cells at any time t ?
- A. $(10,000)2^{0.2t}$
 - B. $(10,000)2^{5t}$
 - C. $(10,000)2^{0.5t}$
 - D. $(10,000)5^{2t}$

FORMULAS

Formula	Description
$C = \pi d$	Circumference of a circle
$A = \pi r^2$	Area of a circle
$V = \frac{1}{3}Bh$	Volume of a pyramid
$V = \frac{1}{3}\pi r^2 h$	Volume of a cone
$V = \pi r^2 h$	Volume of a cylinder
$A = 4\pi r^2$	Surface area of a sphere
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint formula
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$s = r\theta$	Arc length
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
$y = mx + b$	Slope intercept form of line
$a^2 + b^2 = c^2$	Pythagorean theorem
$D = R \cdot T$	Distance, rate, time
$\frac{n!}{r!(n-r)!}$	Combinations
$\frac{n!}{(n-r)!}$	Permutations

FORMULAS (*continued*)

Formula	Description
	Trigonometric ratios
$\sin \theta = \frac{\text{opp}}{\text{hyp}}$	
$\cos \theta = \frac{\text{adj}}{\text{hyp}}$	
$\tan \theta = \frac{\text{opp}}{\text{adj}}$	Formula for infinite geometric series
$S = \frac{A}{1-r}, \text{ where } r < 1$	



Answer Key for the Sample Selected-Response Questions

1. C
2. C
3. B
4. A
5. C
6. B
7. B
8. D
9. D
10. A