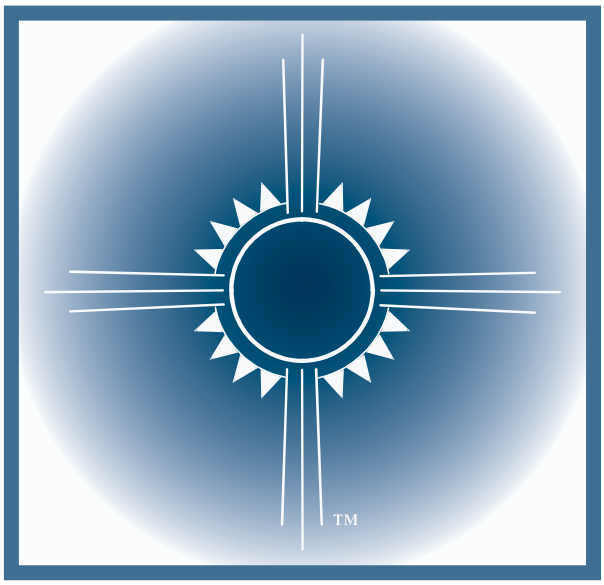


New Mexico

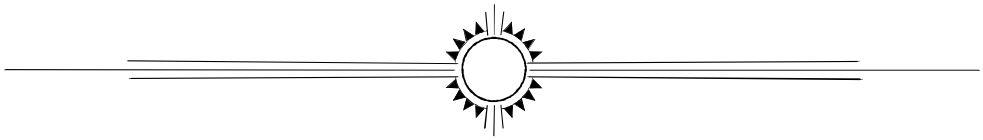
Teacher Assessments™

Study Guide

New Mexico Content Knowledge Assessments™

14 Mathematics

NM-SG-FLD014-03



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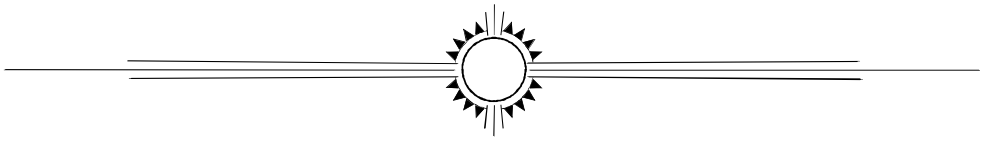
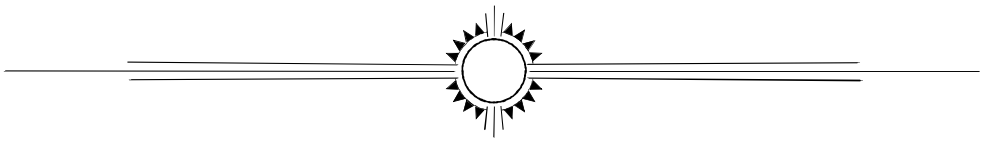


An Overview of the Testing Program, How to Prepare for the Assessments, and the Day of the Test: Helpful Hints

The first three sections of the study guide are available in a separate PDF file. Click the link below to view or print these sections.

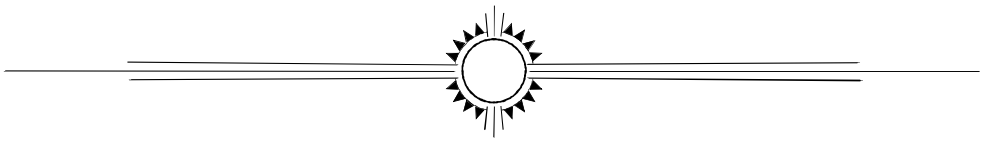
An Overview of the Testing Program, How to Prepare for the Assessments, and the Day of the Test: Helpful Hints



An Introduction to the New Mexico Content Knowledge Assessment of Mathematics

The New Mexico Content Knowledge Assessments are designed to measure the subject matter knowledge and skills that are needed to teach effectively in New Mexico schools. The Mathematics test is intended for individuals seeking a 7–12 teaching license.

The Mathematics test consists of 100 selected-response questions.



Test Framework

Mathematical Processes, Methods, Number Concepts, and Their Historical Development
Geometry and Measurement
Data Analysis, Statistics, Probability, and Discrete Mathematics
Patterns, Algebraic Relationships, and Functions

SUBAREA I—MATHEMATICAL PROCESSES, METHODS, NUMBER CONCEPTS, AND THEIR HISTORICAL DEVELOPMENT

0001 Understands problem-solving strategies, connections among different mathematical ideas, and the use of mathematical modeling to solve real-world problems.

For example:

- evaluates a problem-solving plan, and the reasonableness of a solution to a problem
- applies a range of strategies (e.g., drawing a diagram, working backwards, creating a simpler problem) to solve problems
- selects an appropriate tool or technology to solve a given problem
- recognizes connections among different mathematical concepts (e.g., exploring the relationship between algebra and geometry)
- applies mathematics across the curriculum and in everyday contexts

0002 Understands principles of mathematical reasoning and techniques for communicating mathematical ideas.

For example:

- uses inductive logic to make conjectures and applies deductive logic to develop and evaluate counterexamples and proofs (e.g., direct, indirect)
- interprets and analyzes written mathematical text, symbolism, and concepts
- uses appropriate mathematical terminology and translates common language into symbols and vice versa
- communicates mathematical ideas and concepts using a variety of numeric, symbolic, and graphic methods
- makes connections among numeric, symbolic, graphic, and verbal representations of mathematical ideas and concepts

0003 Understands mathematics as a human endeavor.

For example:

- demonstrates an understanding of the dynamic nature of mathematics and its role in social, cultural, and economic development
- applies knowledge of the historical development of major mathematical concepts, including the importance of the development of mathematical language, numeration systems, and symbolism
- knows the significance of the parallel postulate in the historical development of non-Euclidean geometries
- identifies and analyzes contributions made by various cultures to the growth and development of mathematical ideas
- identifies significant contributions made by individuals in the development of ancient, modern, and current mathematical topics

0004 Understands the appropriate use of technology in the exploration of concepts, skills, and applications in all areas of mathematics.

For example:

- demonstrates an understanding of the use of computing tools such as function graphers, curve fitters, symbolic manipulators, dynamic geometry software, and programming languages to analyze and communicate mathematical information
- recognizes the role of computing tools in solving problems in all areas of mathematics
- demonstrates an understanding of how spreadsheets and statistical packages can be used to analyze and solve problems
- knows how technology is used to explore probability through simulations
- describes how graphing calculators and computer algebra systems can be used in the study and application of calculus
- solves problems using graphing calculators

0005 Understands number systems and equivalent ways of representing numbers.

For example:

- demonstrates an understanding of the use of manipulatives, verbal expressions, and geometric models to represent numbers
- analyzes the role of place value in ancient and modern numeration systems
- identifies and analyzes characteristics and relationships among natural, whole, integer, rational, irrational, real, imaginary, and complex numbers
- uses a variety of equivalent representations of numbers (e.g., fractions, decimals, percents, roots, exponents)
- applies order relations to numbers
- uses scientific notation to solve problems involving very large and very small numbers

0006 Understands number theory and operations on number systems, and extends them to symbolic systems.

For example:

- analyzes properties of prime numbers, factors, multiples, and divisibility
- performs operations with complex numbers (e.g., conjugates, products, roots)
- applies properties of numbers and number operations, including the commutative and distributive properties, to manipulate and simplify algebraic expressions
- uses manipulatives, verbal expressions, and geometric models to represent number operations
- applies and evaluates mental mathematics and estimation strategies, and assesses the reasonableness of a solution to an estimation problem
- analyzes standard and nonstandard computational algorithms
- solves a variety of problems using number operations, including ratio and proportion problems

SUBAREA II—GEOMETRY AND MEASUREMENT

0007 Applies geometric principles of points, lines, angles, planes, congruence, and similarity to analyze the formal characteristics of Euclidean geometry.

For example:

- understands the role of axiomatic systems in geometry and analyzes the relationships among theorems, postulates, definitions, and undefined terms
- determines necessary and sufficient conditions for the existence of a particular shape
- applies properties of parallel and perpendicular lines and angles to analyze shapes
- compares and analyzes shapes and formally establishes the relationships among them (e.g., congruence, similarity)
- applies properties of two-dimensional shapes to analyze three-dimensional shapes
- uses geometric principles to prove theorems

0008 Applies geometric concepts and reasoning as a problem-solving strategy.

For example:

- derives and applies formulas for the perimeter, area, surface area, or volume of two- and three-dimensional composite figures
- explores scale factors for the area and volume of similar figures
- applies right triangle trigonometry and the Pythagorean theorem to solve problems (e.g., problems involving indirect measurements)
- interprets three-dimensional drawings of objects, and analyzes cross sections and nets of three-dimensional figures

0009 Understands coordinate, vector, and transformational geometry.

For example:

- uses a coordinate system to locate and describe the locus of points that satisfy a given condition
- locates objects in terms of their position using rectangular, polar, and three-dimensional coordinate systems
- applies concepts of slope, distance, midpoint, and parallel and perpendicular lines to determine the geometric and algebraic properties of figures in the coordinate plane, including conic sections
- describes the position and movement of objects using vectors
- analyzes figures in terms of translations, reflections, rotations, dilations, and contractions
- applies transformations (e.g., translations, reflections, rotations, dilations, contractions) to explore the concepts of congruence and similarity, and to characterize the symmetry of an object

0010 Understands and uses measurement.

For example:

- selects and uses appropriate units of measurement for angles (e.g., degree, radian), length, area, volume, mass, temperature, and time
- uses standard and nonstandard units of measurement to an appropriate degree of accuracy
- applies concepts of precision and accuracy to evaluate measurements and measurement error
- identifies appropriate tools for performing measurements
- converts measurements within and between traditional and metric measuring systems
- uses dimensional analysis to solve problems
- solves a variety of problems in mathematics and other disciplines involving linear measures, area, volume, mass, and temperature, as well as derived units such as density, pressure, and rates of change

SUBAREA III—DATA ANALYSIS, STATISTICS, PROBABILITY, AND DISCRETE MATHEMATICS

0011 Understands methods of collecting, organizing, displaying, describing, and analyzing data.

For example:

- formulates questions and collects data using appropriate techniques in a variety of situations, and recognizes bias factors that may affect the validity of a survey
- uses a variety of charts and graphs to organize and display data (e.g., pie charts, box plots, stem-and-leaf plots, scatter plots, frequency histograms) appropriately
- applies and interprets measures of central tendency (e.g., mean, median, mode) and spread (e.g., range, standard deviation)
- recognizes the effects of data transformations on central tendency and spread
- finds a function (e.g., linear, logarithmic, exponential) that best represents a set of data

0012 Understands data, and making predictions and inferences based on data.

For example:

- interprets and explains sample data, and determines what inferences can be drawn from a study
- evaluates a statistical argument and analyzes factors (e.g., sampling technique, sample size, width of confidence intervals) that may affect its validity
- applies the principles of interpolation and extrapolation to make predictions based on data
- analyzes linear regression lines and correlation coefficients
- makes decisions based on uncertainty using a variety of approaches (e.g., hypothesis testing, simulations, analysis of variance, non-parametric methods)

0013 Understands the theory of probability and probability distributions.

For example:

- enumerates the sample space of an event, and determines simple and compound probabilities
- finds the probability of dependent and independent events, and determines conditional probabilities
- uses simulations and sampling to determine experimental probabilities
- distinguishes between theoretical probability and experimental outcomes
- calculates expected values of a probability distribution
- solves problems using geometric probability (e.g., ratio of two areas)
- uses probability concepts and distributions (e.g., binomial, normal) to model and solve a variety of real-world problems

0014 Understands and applies principles of discrete mathematics.

For example:

- solves counting problems using permutations and combinations
- uses sets and set relations to represent algebraic and geometric concepts
- uses finite graphs and trees to solve network problems (e.g., finding circuits and critical paths)
- employs recursion and iteration methods to model problems
- describes and analyzes efficient algorithms to accomplish a task or solve a problem in a variety of contexts (e.g., practical and computer-related situations)
- uses linear programming to model and solve problems
- uses finite difference equations to model and solve problems

SUBAREA IV—PATTERNS, ALGEBRAIC RELATIONSHIPS, AND FUNCTIONS

0015 Describes, analyzes, and generalizes mathematical patterns.

For example:

- recognizes, describes, and extends a variety of numerical and geometric patterns
- represents and records patterns using manipulatives, tables, graphs, and matrices
- explores and describes symmetric and spatial patterns (e.g., fractals, tessellations)
- analyzes and generalizes sequences and series (e.g., Fibonacci, geometric) and uses them to solve problems
- recognizes and extends recursive patterns, and uses them to solve problems
- uses patterns to make inferences, predictions, and decisions

0016 Uses variables and symbolic expressions to describe and analyze patterns of change, functions, and algebraic relationships.

For example:

- represents situations using variables and expressions
- explores patterns of change characteristic of families of functions (e.g., linear, quadratic, exponential)
- translates among verbal, graphic, tabular, and symbolic representations of functions in mathematics and other disciplines
- distinguishes between relations and functions
- analyzes functions in terms of range, domain, and intercepts
- explores functions in terms of operations, compositions, and inverses
- analyzes the relationship among the graphs of $f(x)$ and transformations
  [e.g., $f(x \pm c)$, $f(x) \pm c$, $cf(x)$, $\frac{1}{f(x)}$]
- applies the major concepts of linear and abstract algebra (e.g., matrices, vectors, groups)

0017 Understands properties and applications of linear functions and solves related equations and inequalities.

For example:

- analyzes the relationship between a linear equation and its graph, including its slope and intercepts
- determines the equation of a line in a variety of situations
- models and solves problems using linear equations and inequalities
- predicts or interpolates values using a linear model
- evaluates the accuracy and appropriateness of a linear model in representing a particular situation
- models and solves linear systems using a variety of methods (e.g., substitution, graphs, matrices)

0018 Understands properties and applications of quadratic and higher order polynomial functions, and solves related equations and inequalities.

For example:

- analyzes the roots of a quadratic equation
- analyzes how changing the coefficients of a quadratic equation changes its graph
- solves quadratic equations, inequalities, and systems using a variety of methods (e.g., graphical, analytical)
- analyzes polynomial functions and their graphs (e.g., zeros, local minima and maxima, inflection points, end behaviors, symmetry)
- uses quadratic and polynomial functions to model and solve problems, including maximum and minimum problems

0019 Understands properties and applications of rational, radical, exponential, logarithmic, and trigonometric functions, and solves related equations and inequalities.

For example:

- analyzes the relationship between the properties of rational and radical functions and their graphs (e.g., asymptotes, discontinuities)
- analyzes and solves problems (e.g., growth and decay) involving the relationship between exponential and logarithmic functions
- analyzes and applies the relationship between inverse variation and rational functions
- examines the relationship between the trigonometric functions and their inverses, and between the trigonometric and circular functions
- applies these functions to model and solve problems involving real-world situations (e.g., periodic phenomena, Richter scale measurements)

0020 Understands principles and applications of differential and integral calculus.

For example:

- investigates the limits of sequences and series
- analyzes rate of change as a limiting process, and recognizes the characteristics of the difference quotient
- uses limits to determine the continuity of a function
- analyzes the relationships among the graph, slope of the secant line, and the derivative of a function
- uses differential calculus to analyze the graph of a function
- analyzes the relationship between the area under a curve, Riemann sums, and integration
- uses the relationship between the derivative and area under the curve to solve a variety of problems
- uses calculus to model and solve a variety of real-world problems (e.g., average and instantaneous rates of change, area, volume, curve length)

Sample Test Directions

A sample of the general directions for the New Mexico Content Knowledge Assessment of Mathematics is shown in the box below.

You should have in front of you:

(1) a test booklet for the assessment for which you registered (check the assessment name on the front cover);

(2) an Answer Sheet A; and

(3) a No. 2 lead pencil.

Note: For the **Mathematics (14)** test only, you may use your own calculator if it is an approved model.

IF YOU ARE MISSING ANY OF THESE MATERIALS, NOTIFY YOUR TEST ADMINISTRATOR. REMOVE ALL OTHER MATERIALS FROM YOUR DESK.

TEST DIRECTIONS

Each question in this booklet is a selected-response question with four answer choices. Read each question carefully and choose the ONE best answer. Record your answer on Answer Sheet A in the space that corresponds to the question number. Completely fill in the circle having the same letter as the answer you have chosen. *Use only a No. 2 lead pencil.*

Sample Question: 1. What is the capital of New Mexico?

A. Albuquerque
B. Las Cruces
C. Santa Fe
D. Silver City

The correct answer to this question is C. You would indicate that on Answer Sheet A as follows:

1. Ⓐ Ⓑ ● Ⓓ

Try to answer all questions. Even if you are unsure of an answer, it is better to guess than not to answer a question at all. You will NOT be penalized for guessing.

You may use the margins of the test booklet for scratch paper, but all of your answers must be recorded on the answer sheet. Answers that are in the test booklet will not be scored.

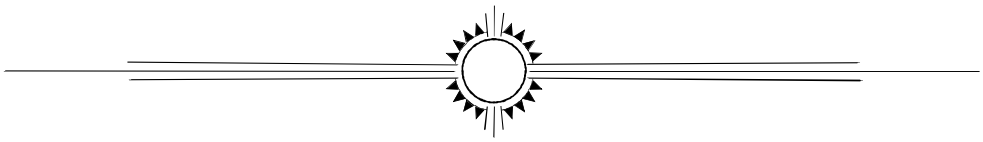
The words "End of Test" indicate that you have completed the test. You may go back and review your work, and be sure you have answered all questions before raising your hand for dismissal. Your test materials must be returned to a test administrator when you finish the test.

FOR TEST SECURITY REASONS, YOU MAY NOT TAKE NOTES OR REMOVE ANY OF THE TEST MATERIALS FROM THE ROOM.

This testing session will last four hours. You may work at your own pace. If you have any questions, please ask them now before beginning the test.

STOP

DO NOT GO ON UNTIL YOU ARE TOLD TO DO SO.



Sample Selected-Response Questions

This section presents sample selected-response questions for you to review as part of your preparation for the New Mexico Content Knowledge Assessment of Mathematics. To demonstrate how the test competencies may be assessed, each sample question is preceded by the competency that it measures. On an actual test, the competencies will not be given.

The sample selected-response questions are designed to illustrate the nature of the test questions. They should not be used as a diagnostic tool to determine your individual strengths and weaknesses. The selected-response questions require you to demonstrate more than the ability to recall factual information. They ask you to think critically about the information, to analyze it, to consider it carefully, or to apply it to a hypothetical situation.

Work through each question carefully before referring to the answer key, which is located at the end of the section.

Competency 0002
Understands principles of mathematical reasoning and techniques for communicating mathematical ideas.

1. Sixth-grade students are using a graphing calculator to explore how changing the value of *m* in an equation of the form $y = mx$ changes the graph of the equation. The students view several graphs and are then asked to make a generalization about how the value of *m* affects the graph of the equation. This is an example of using:

   A. a counterexample to evaluate a mathematical relationship.

   B. an axiomatic system to generate a mathematical relationship.

   C. inductive reasoning to conjecture a mathematical relationship.

   D. deductive reasoning to prove a mathematical relationship.

Competency 0003
Understands mathematics as a human endeavor.

2. An ancient Chinese text dating from around 250 B.C., *The Nine Chapters on the Mathematical Art,* contained 246 solved mathematical problems on a variety of topics. One problem is given below.

   > A circular pond has a radius of 5 units. How deep is the pond if a reed growing in the center and extending 1 unit above the water just reaches the surface if drawn to the edge of the pond?

   The ability to solve this problem indicates that ancient Chinese mathematicians were able to:

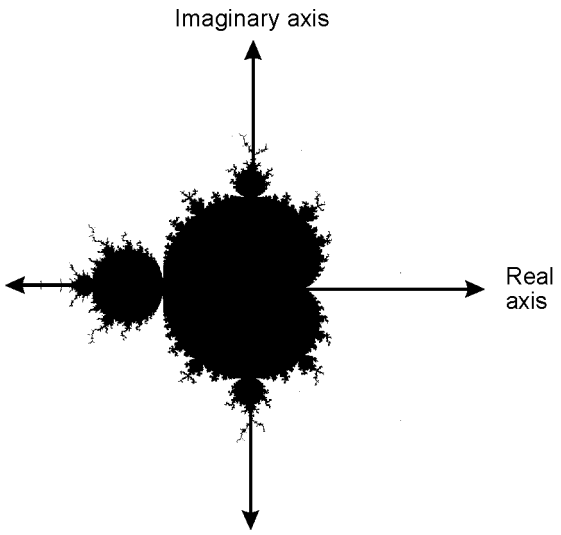
   A. solve two linear equations in two unknowns.

   B. apply the principles of trigonometry.

   C. solve a quadratic equation for positive roots.

   D. apply the Pythagorean theorem.

3. Use the diagram below to answer the question that follows.



Given that a complex number $z = a + bi$ is a point in the shaded region of the complex plane in the diagram above, which of the following must also be a point in the shaded region?

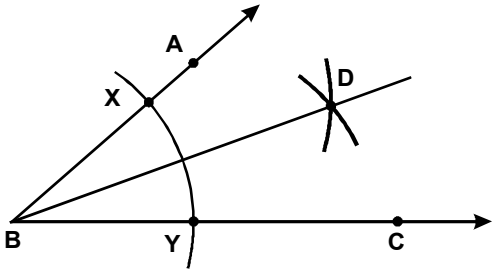
A. $a - bi$

B. $-b + ai$

C. $\frac{1}{a + bi}$

D. $(a + bi)^2$

4. Use the diagram below to answer the question that follows.



Given $\angle ABC$, a student places the point of a compass at B and draws an arc intersecting $\overrightarrow{BA}$ and $\overrightarrow{BC}$ at X and Y, respectively. The student next places the compass point on X and Y and draws two arcs of equal radius intersecting at D. The student draws $\overrightarrow{BD}$, and wants to prove that $\triangle BXD \cong \triangle BYD$ to show that $\overrightarrow{BD}$ bisects $\angle B$. Which of the following can be used to prove the triangles congruent?

A. SSS

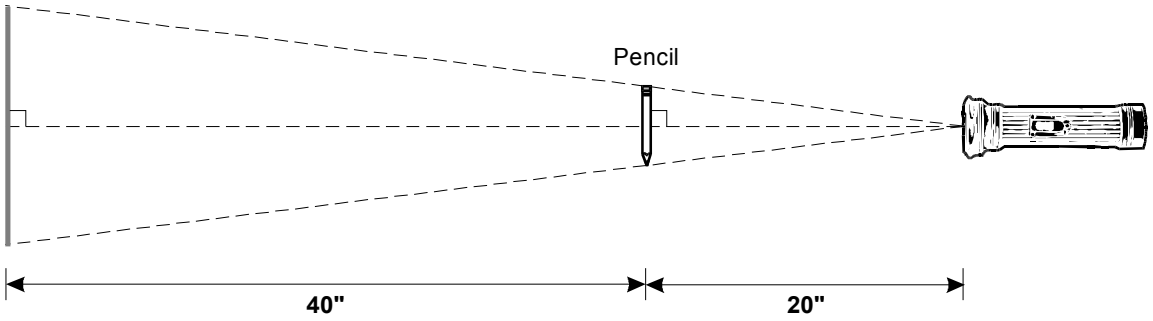
B. ASA

C. SAS

D. AAA

Competency 0007
Applies geometric principles of points, lines, angles, planes, congruence, and similarity to analyze the formal characteristics of Euclidean geometry.

5. Use the diagram below to answer the question that follows.



A pencil 5 inches long is held between a wall and a flashlight creating a shadow on the wall as shown above. What is the length of the pencil's shadow in inches?

A. 10

B. 12

C. 15

D. 20

Competency 0011
Understands methods of collecting, organizing, displaying, describing, and analyzing data.

6. A student makes seven measurements, rejects the lowest and highest, and uses the remaining five to do some statistical analyses. Which of the following will remain unaffected by rejecting the lowest and highest measurements?

A. mean

B. median

C. range

D. standard deviation

Competency 0013
Understands the theory of probability and probability distributions.

7. A high school gym class consists of 5 seniors, 7 juniors, 8 sophomores, and 6 freshmen. Two students are chosen at random to assist the teacher with setting up equipment. What is the probability that a freshman will be chosen first and a senior chosen second?

   A. $\frac{2}{26}$

   B. $\frac{11}{26}$

   C. $\frac{30}{650}$

   D. $\frac{30}{676}$

Competency 0018
Understands properties and applications of quadratic and higher order polynomial functions, and solves related equations and inequalities.

8. A quadratic function h(x) has zeros at 4 and –3 and a y-intercept of –12. The function h(x) is translated –3 units on the x-axis. Which of the following equations represents g(x), the transformation of h(x)?

   A. $g(x) = x^2 - 5x$

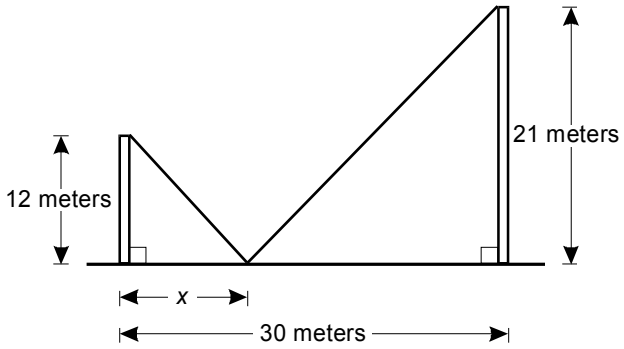
   B. $g(x) = x^2 + 7x$

   C. $g(x) = x^2 - x - 15$

   D. $g(x) = x^2 + 5x - 6$

Competency 0019
Understands properties and applications of rational, radical, exponential, logarithmic, and trigonometric functions, and solves related equations and inequalities.

9. Use the diagram below to answer the question that follows.



Two posts, one 12 meters high and the other 21 meters high, are 30 meters apart. A single wire runs from the top of the first post to the ground and from the ground to the top of the second post. The wire is attached to the ground at a distance of x units from the first post. Which of the following functions represents the total length of the wire?

A. $\ell(x) = \sqrt{1485 - 60x}$

B. $\ell(x) = \sqrt{1485 - 60x + 2x^2}$

C. $\ell(x) = \sqrt{144 + x^2} + \sqrt{1341 + x^2}$

D. $\ell(x) = \sqrt{144 + x^2} + \sqrt{1341 - 60x + x^2}$

Competency 0020
Understands principles and applications of differential and integral calculus.

10. A company that manufactures air conditioners finds that its profit P(x) from selling x units can be modeled by the equation $P(x) = 25\sqrt{x} - 0.05x - 250$. A company analyst wishes to find the average rate of change of the profit function between selling 100 units and 200 units. Which of the following procedures should the analyst use?

    A. Find the slope of the secant line between those two points on the graph of P(x).

    B. Find the value of the derivative of the function at those two points and average them.

    C. Find the area under the graph of P(x) between the two points.

    D. Find the value of P(x) at those two points and average them.

FORMULAS

Formula	Description
$V = \frac{1}{3}Bh$	Volume of a pyramid
$V = \frac{1}{3}\pi r^2 h$	Volume of a cone
$V = \pi r^2 h$	Volume of a cylinder
$A = 4\pi r^2$	Surface area of a sphere
$V = \frac{4}{3}\pi r^3$	Volume of a sphere
$S_n = \frac{n}{2}[2a + (n-1)d] = n\left(\frac{a + a_n}{2}\right)$	Sum of an arithmetic series
$S_n = \frac{a(1 - r^n)}{1 - r}$	Sum of a geometric series
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance formula
$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$	Midpoint formula
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$	Slope
$s = r\theta$	Arc length
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Quadratic formula
$\frac{n!}{r!(n-r)!}$	Combinations
$\frac{n!}{(n-r)!}$	Permutations

FORMULAS (continued)

Formula	Description
$\sin \theta = \frac{opp}{hyp}$	Sine
$\cos \theta = \frac{adj}{hyp}$	Cosine
$\tan \theta = \frac{opp}{adj}$	Tangent

Answer Key for the Sample Selected-Response Questions

1. C
2. D
3. A
4. A
5. C
6. B
7. C
8. D
9. D
10. A