

# NEW MEXICO TEACHER ASSESSMENTS™

## TEST FRAMEWORK

### FIELD 14: MATHEMATICS

Subarea		Range of Competencies	Test Proportion
I.	Mathematical Processes, Methods, Number Concepts, and Their Historical Development	0001–0006	30%
II.	Geometry and Measurement	0007–0010	20%
III.	Data Analysis, Statistics, Probability, and Discrete Mathematics	0011–0014	20%
IV.	Patterns, Algebraic Relationships, and Functions	0015–0020	30%

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# NEW MEXICO TEACHER ASSESSMENTS™

## Field 14: Mathematics

Mathematical Processes, Methods, Number Concepts, and Their Historical Development  
Geometry and Measurement  
Data Analysis, Statistics, Probability, and Discrete Mathematics  
Patterns, Algebraic Relationships, and Functions

### SUBAREA I—MATHEMATICAL PROCESSES, METHODS, NUMBER CONCEPTS, AND THEIR HISTORICAL DEVELOPMENT

**0001 Understands problem-solving strategies, connections among different mathematical ideas, and the use of mathematical modeling to solve real-world problems.**

For example:

- evaluates a problem-solving plan, and the reasonableness of a solution to a problem
- applies a range of strategies (e.g., drawing a diagram, working backwards, creating a simpler problem) to solve problems
- selects an appropriate tool or technology to solve a given problem
- recognizes connections among different mathematical concepts (e.g., exploring the relationship between algebra and geometry)
- applies mathematics across the curriculum and in everyday contexts

**0002 Understands principles of mathematical reasoning and techniques for communicating mathematical ideas.**

For example:

- uses inductive logic to make conjectures and applies deductive logic to develop and evaluate counterexamples and proofs (e.g., direct, indirect)
- interprets and analyzes written mathematical text, symbolism, and concepts
- uses appropriate mathematical terminology and translates common language into symbols and vice versa
- communicates mathematical ideas and concepts using a variety of numeric, symbolic, and graphic methods
- makes connections among numeric, symbolic, graphic, and verbal representations of mathematical ideas and concepts

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### **0003 Understands mathematics as a human endeavor.**

For example:

- demonstrates an understanding of the dynamic nature of mathematics and its role in social, cultural, and economic development
- applies knowledge of the historical development of major mathematical concepts, including the importance of the development of mathematical language, numeration systems, and symbolism
- knows the significance of the parallel postulate in the historical development of non-Euclidean geometries
- identifies and analyzes contributions made by various cultures to the growth and development of mathematical ideas
- identifies significant contributions made by individuals in the development of ancient, modern, and current mathematical topics

### **0004 Understands the appropriate use of technology in the exploration of concepts, skills, and applications in all areas of mathematics.**

For example:

- demonstrates an understanding of the use of computing tools such as function graphers, curve fitters, symbolic manipulators, dynamic geometry software, and programming languages to analyze and communicate mathematical information
- recognizes the role of computing tools in solving problems in all areas of mathematics
- demonstrates an understanding of how spreadsheets and statistical packages can be used to analyze and solve problems
- knows how technology is used to explore probability through simulations
- describes how graphing calculators and computer algebra systems can be used in the study and application of calculus
- solves problems using graphing calculators

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**0005 Understands number systems and equivalent ways of representing numbers.**

For example:

- demonstrates an understanding of the use of manipulatives, verbal expressions, and geometric models to represent numbers
- analyzes the role of place value in ancient and modern numeration systems
- identifies and analyzes characteristics and relationships among natural, whole, integer, rational, irrational, real, imaginary, and complex numbers
- uses a variety of equivalent representations of numbers (e.g., fractions, decimals, percents, roots, exponents)
- applies order relations to numbers
- uses scientific notation to solve problems involving very large and very small numbers

**0006 Understands number theory and operations on number systems, and extends them to symbolic systems.**

For example:

- analyzes properties of prime numbers, factors, multiples, and divisibility
- performs operations with complex numbers (e.g., conjugates, products, roots)
- applies properties of numbers and number operations, including the commutative and distributive properties, to manipulate and simplify algebraic expressions
- uses manipulatives, verbal expressions, and geometric models to represent number operations
- applies and evaluates mental mathematics and estimation strategies, and assesses the reasonableness of a solution to an estimation problem
- analyzes standard and nonstandard computational algorithms
- solves a variety of problems using number operations, including ratio and proportion problems

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**SUBAREA II—GEOMETRY AND MEASUREMENT**

**0007 Applies geometric principles of points, lines, angles, planes, congruence, and similarity to analyze the formal characteristics of Euclidean geometry.**

For example:

- understands the role of axiomatic systems in geometry and analyzes the relationships among theorems, postulates, definitions, and undefined terms
- determines necessary and sufficient conditions for the existence of a particular shape
- applies properties of parallel and perpendicular lines and angles to analyze shapes
- compares and analyzes shapes and formally establishes the relationships among them (e.g., congruence, similarity)
- applies properties of two-dimensional shapes to analyze three-dimensional shapes
- uses geometric principles to prove theorems

**0008 Applies geometric concepts and reasoning as a problem-solving strategy.**

For example:

- derives and applies formulas for the perimeter, area, surface area, or volume of two- and three-dimensional composite figures
- explores scale factors for the area and volume of similar figures
- applies right triangle trigonometry and the Pythagorean theorem to solve problems (e.g., problems involving indirect measurements)
- interprets three-dimensional drawings of objects, and analyzes cross sections and nets of three-dimensional figures

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**0009 Understands coordinate, vector, and transformational geometry.**

For example:

- uses a coordinate system to locate and describe the locus of points that satisfy a given condition
- locates objects in terms of their position using rectangular, polar, and three-dimensional coordinate systems
- applies concepts of slope, distance, midpoint, and parallel and perpendicular lines to determine the geometric and algebraic properties of figures in the coordinate plane, including conic sections
- describes the position and movement of objects using vectors
- analyzes figures in terms of translations, reflections, rotations, dilations, and contractions
- applies transformations (e.g., translations, reflections, rotations, dilations, contractions) to explore the concepts of congruence and similarity, and to characterize the symmetry of an object

**0010 Understands and uses measurement.**

For example:

- selects and uses appropriate units of measurement for angles (e.g., degree, radian), length, area, volume, mass, temperature, and time
- uses standard and nonstandard units of measurement to an appropriate degree of accuracy
- applies concepts of precision and accuracy to evaluate measurements and measurement error
- identifies appropriate tools for performing measurements
- converts measurements within and between traditional and metric measuring systems
- uses dimensional analysis to solve problems
- solves a variety of problems in mathematics and other disciplines involving linear measures, area, volume, mass, and temperature, as well as derived units such as density, pressure, and rates of change

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**SUBAREA III—DATA ANALYSIS, STATISTICS, PROBABILITY, AND DISCRETE MATHEMATICS**

**0011 Understands methods of collecting, organizing, displaying, describing, and analyzing data.**

For example:

- formulates questions and collects data using appropriate techniques in a variety of situations, and recognizes bias factors that may affect the validity of a survey
- uses a variety of charts and graphs to organize and display data (e.g., pie charts, box plots, stem-and-leaf plots, scatter plots, frequency histograms) appropriately
- applies and interprets measures of central tendency (e.g., mean, median, mode) and spread (e.g., range, standard deviation)
- recognizes the effects of data transformations on central tendency and spread
- finds a function (e.g., linear, logarithmic, exponential) that best represents a set of data

**0012 Understands data, and making predictions and inferences based on data.**

For example:

- interprets and explains sample data, and determines what inferences can be drawn from a study
- evaluates a statistical argument and analyzes factors (e.g., sampling technique, sample size, width of confidence intervals) that may affect its validity
- applies the principles of interpolation and extrapolation to make predictions based on data
- analyzes linear regression lines and correlation coefficients
- makes decisions based on uncertainty using a variety of approaches (e.g., hypothesis testing, simulations, analysis of variance, non-parametric methods)

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**0013 Understand the theory of probability and probability distributions.**

For example:

- enumerates the sample space of an event, and determines simple and compound probabilities
- finds the probability of dependent and independent events, and determines conditional probabilities
- uses simulations and sampling to determine experimental probabilities
- distinguishes between theoretical probability and experimental outcomes
- calculates expected values of a probability distribution
- solves problems using geometric probability (e.g., ratio of two areas)
- uses probability concepts and distributions (e.g., binomial, normal) to model and solve a variety of real-world problems

**0014 Understands and applies principles of discrete mathematics.**

For example:

- solves counting problems using permutations and combinations
- uses sets and set relations to represent algebraic and geometric concepts
- uses finite graphs and trees to solve network problems (e.g., finding circuits and critical paths)
- employs recursion and iteration methods to model problems
- describes and analyzes efficient algorithms to accomplish a task or solve a problem in a variety of contexts (e.g., practical and computer-related situations)
- uses linear programming to model and solve problems
- uses finite difference equations to model and solve problems

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**SUBAREA IV—PATTERNS, ALGEBRAIC RELATIONSHIPS, AND FUNCTIONS**

**0015 Describes, analyzes, and generalizes mathematical patterns.**

For example:

- recognizes, describes, and extends a variety of numerical and geometric patterns
- represents and records patterns using manipulatives, tables, graphs, and matrices
- explores and describes symmetric and spatial patterns (e.g., fractals, tessellations)
- analyzes and generalizes sequences and series (e.g., Fibonacci, geometric) and uses them to solve problems
- recognizes and extends recursive patterns, and uses them to solve problems
- uses patterns to make inferences, predictions, and decisions

**0016 Uses variables and symbolic expressions to describe and analyze patterns of change, functions, and algebraic relationships.**

For example:

- represents situations using variables and expressions
- explores patterns of change characteristic of families of functions (e.g., linear, quadratic, exponential)
- translates among verbal, graphic, tabular, and symbolic representations of functions in mathematics and other disciplines
- distinguishes between relations and functions
- analyzes functions in terms of range, domain, and intercepts
- explores functions in terms of operations, compositions, and inverses
- analyzes the relationship among the graphs of  $f(x)$  and transformations [e.g.,  $f(x \pm c)$ ,  $f(x) \pm c$ ,  $cf(x)$ ,  $\frac{1}{f(x)}$ ]
- applies the major concepts of linear and abstract algebra (e.g., matrices, vectors, groups)

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**0017 Understands properties and applications of linear functions and solves related equations and inequalities.**

For example:

- analyzes the relationship between a linear equation and its graph, including its slope and intercepts
- determines the equation of a line in a variety of situations
- models and solves problems using linear equations and inequalities
- predicts or interpolates values using a linear model
- evaluates the accuracy and appropriateness of a linear model in representing a particular situation
- models and solves linear systems using a variety of methods (e.g., substitution, graphs, matrices)

**0018 Understands properties and applications of quadratic and higher order polynomial functions, and solves related equations and inequalities.**

For example:

- analyzes the roots of a quadratic equation
- analyzes how changing the coefficients of a quadratic equation changes its graph
- solves quadratic equations, inequalities, and systems using a variety of methods (e.g., graphical, analytical)
- analyzes polynomial functions and their graphs (e.g., zeros, local minima and maxima, inflection points, end behaviors, symmetry)
- uses quadratic and polynomial functions to model and solve problems, including maximum and minimum problems

**0019 Understands properties and applications of rational, radical, exponential, logarithmic, and trigonometric functions, and solves related equations and inequalities.**

For example:

- analyzes the relationship between the properties of rational and radical functions and their graphs (e.g., asymptotes, discontinuities)
- analyzes and solves problems (e.g., growth and decay) involving the relationship between exponential and logarithmic functions
- analyzes and applies the relationship between inverse variation and rational functions
- examines the relationship between the trigonometric functions and their inverses, and between the trigonometric and circular functions
- applies these functions to model and solve problems involving real-world situations (e.g., periodic phenomena, Richter scale measurements)

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**0020 Understands principles and applications of differential and integral calculus.**

For example:

- investigates the limits of sequences and series
- analyzes rate of change as a limiting process, and recognizes the characteristics of the difference quotient
- uses limits to determine the continuity of a function
- analyzes the relationships among the graph, slope of the secant line, and the derivative of a function
- uses differential calculus to analyze the graph of a function
- analyzes the relationship between the area under a curve, Riemann sums, and integration
- uses the relationship between the derivative and area under the curve to solve a variety of problems
- uses calculus to model and solve a variety of real-world problems (e.g., average and instantaneous rates of change, area, volume, curve length)